Diffractive parton distributions: the role of the perturbative Pomeron

G. Watt[1], *A.D. Martin*[2] *and M.G. Ryskin*[3] †
[1] Department of Physics & Astronomy, University College London, WC1E 6BT, UK
[2] Institute for Particle Physics Phenomenology, University of Durham, DH1 3LE, UK
[3] Petersburg Nuclear Physics Institute, Gatchina, St. Petersburg, 188300, Russia

Abstract
We consider the role of the perturbative Pomeron-to-parton splitting in the formation of the diffractive parton distributions.

Diffractive deep-inelastic scattering (DDIS), $\gamma^* p \to X + p$, is characterised by a large rapidity gap (LRG) between the cluster X of outgoing hadrons and the slightly deflected proton, understood to be due to 'Pomeron' exchange. Let the momenta of the incoming proton, the outgoing proton, and the photon be labelled p, p' and q respectively; see Fig. 1(a). Then the basic kinematic variables in DDIS are the photon virtuality, $Q^2 = -q^2$, the Bjorken-x variable, $x_B = Q^2/(2p \cdot q)$, the squared momentum transfer, $t = (p - p')^2$, the fraction of the proton's light-cone momentum transferred through the LRG, $x_{\mathbb{P}} = 1 - p'^+/p^+$, and $\beta \equiv x_B/x_{\mathbb{P}}$.

It is common to perform analyses of DDIS data based on two levels of factorisation. First the t-integrated diffractive structure function $F_2^{\mathrm{D}(3)}$ may be written as the convolution of the usual coefficient functions $C_{2,a}$ as in DIS with diffractive parton distribution functions (DPDFs) a^{D} [1]:

$$F_2^{\mathrm{D}(3)}(x_{\mathbb{P}}, \beta, Q^2) \;=\; \sum_{a=q,g} \beta \int_\beta^1 \frac{\mathrm{d}z}{z^2}\, C_{2,a}\left(\frac{\beta}{z}\right)\, a^{\mathrm{D}}(x_{\mathbb{P}}, z, \mu_F^2), \tag{1}$$

with the factorisation scale μ_F usually taken to be Q. The DPDFs $a^{\mathrm{D}} = zq^{\mathrm{D}}$ or zg^{D} satisfy DGLAP evolution in μ_F. The convolution variable $z \in [\beta, 1]$ is the fraction of the Pomeron's light-cone momentum carried by the parton entering the hard subprocess. At leading-order (LO) the coefficient functions are $C_{2,q}(x) = e_q^2\,\delta(1-x)$ and $C_{2,g}(x) = 0$. The collinear factorisation theorem (1) applies when μ_F is made very large; it is correct up to power-suppressed corrections. In the second stage, Regge factorisation is usually assumed, such that the diffractive parton densities a^{D} are written as the product of the Pomeron flux factor $f_{\mathbb{P}} = \int \mathrm{d}t\; \exp(B_{\mathbb{P}}\,t)\; x_{\mathbb{P}}^{1-2\alpha_{\mathbb{P}}(t)}$ and the Pomeron parton densities $a^{\mathbb{P}} = zq^{\mathbb{P}}$ or $zg^{\mathbb{P}}$, that is,

$$a^{\mathrm{D}}(x_{\mathbb{P}}, z, \mu_F^2) = f_{\mathbb{P}}(x_{\mathbb{P}})\, a^{\mathbb{P}}(z, \mu_F^2). \tag{2}$$

The Pomeron trajectory is $\alpha_{\mathbb{P}}(t) = \alpha_{\mathbb{P}}(0) + \alpha'_{\mathbb{P}}\,t$. For simplicity, we omit the contribution of secondary Reggeons to the right-hand side of (2). Such an approach says nothing about the mechanism for diffraction: information about the diffractive exchange ('Pomeron') needs to be parameterised at the input scale μ_0 and fitted to the data.

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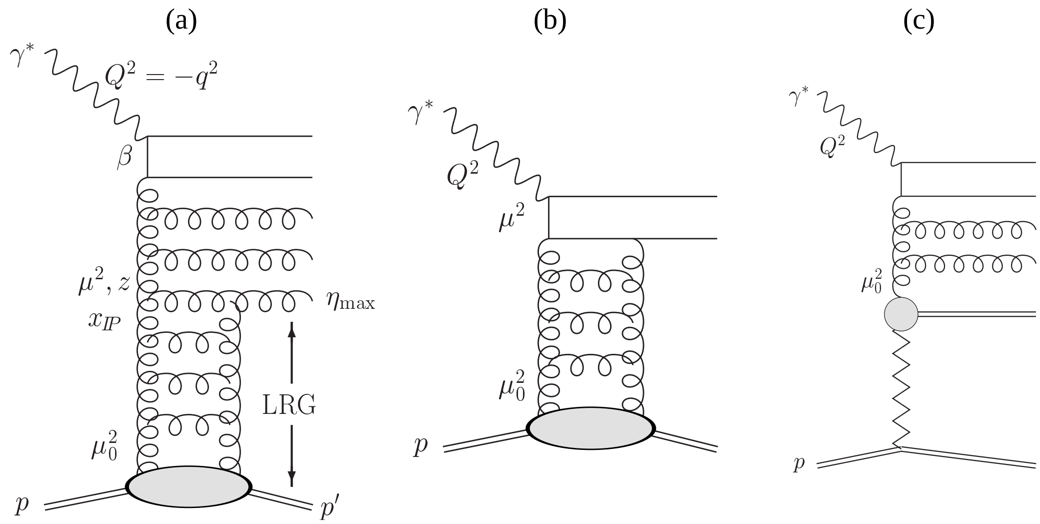

Fig. 1: (a) The perturbative *resolved* Pomeron contribution, which is the basis of the perturbative QCD approach, (b) the perturbative *direct* Pomeron contribution, and (c) the non-perturbative *resolved* Pomeron diagram, which accounts for the contribution from low scales, $\mu < \mu_0$.

An alternative way to describe DDIS is to consider the heavy photon transition to $q\bar{q}$ or effective $(q\bar{q})g$ dipoles, which then interact with the target proton via two-gluon exchange [2, 3]; the case of the $q\bar{q}$ dipole is shown in Fig. 1(b). Here the Pomeron is modelled by the t-channel colour-singlet pair of gluons and we have an explicit form of the Pomeron parton distributions $a^{\mathbb{P}}(z, \mu^2)$ given at the initial virtuality $\mu^2 = k_t^2/(1-z)$ fixed by the transverse momentum k_t of the outgoing components of the dipole. Such an approach relies on the existence of a large saturation scale $Q_S^2 \gtrsim 1$ GeV^2 [4] to act as an infrared cutoff and suppress the contribution from large dipole sizes, thereby justifying the use of perturbative QCD for the whole $F_2^{\mathrm{D}(3)}$. However, more sophisticated dipole models [5] now give a lower $Q_S^2 \lesssim 0.5$ GeV^2 in the HERA kinematic regime, therefore significant non-perturbative contributions to inclusive DDIS are to be expected.

The two theoretical frameworks are essentially contradictory: the Regge factorisation approach is motivated by 'soft' Pomeron exchange, whereas the two-gluon exchange approach is motivated by perturbative QCD. Nevertheless, a 'doublethink' mentality exists whereby the two approaches are both commonly applied separately in the description of data, often within the same paper, with few attempts made to reconcile them.

The two approaches can be combined [6–8] by generalising the $\gamma^* \to q\bar{q}$ and $\gamma^* \to q\bar{q}g$ transitions to an arbitrary number of parton emissions in the final state, as shown in the upper half of Fig. 1(a). The perturbative Pomeron is described by a parton ladder ending in a pair of t-channel gluons or sea-quarks; the former is shown in the lower half of Fig. 1(a). The virtualities of the t-channel partons are strongly ordered as required by DGLAP evolution: $\mu_0^2 \ll \ldots \ll \mu^2 \ll \ldots \ll \mu_F^2$. The scale μ^2 at which the Pomeron-to-parton splitting occurs can vary between $\mu_0^2 \sim 1$ GeV^2 and μ_F^2. For $\mu < \mu_0$, the representation of the Pomeron as a perturbative parton ladder is no longer valid, and instead, in the lack of a precise theory of non-perturbative QCD, we appeal to Regge theory where the 'soft' Pomeron is treated as an effective Regge pole with

intercept $\alpha_{\mathbb{P}}(0) \simeq 1.08$; see Fig. 1(c).

The probability to find an appropriate pair of t-channel gluons with transverse momentum l_t, integrated over l_t, is given by the usual gluon distribution of the proton obtained from the global parton analyses, $x_{\mathbb{P}} g(x_{\mathbb{P}}, \mu^2)$, where $\mu^2 = k_t^2/(1-z)$ is the virtuality of the first t-channel parton in the upper part of the diagram. The emitted parton at the edge of the LRG in Fig. 1(a) has rapidity $\eta_{\rm max}$ and transverse momentum k_t and carries a fraction $(1-z)$ of the Pomeron's light-cone momentum. For inclusive DDIS we must integrate over k_t, accounting for the components of the Pomeron wave function of different sizes $\sim 1/k_t$, which translates to an integral over μ of the form

$$a^{\rm D}(x_{\mathbb{P}}, z, \mu_F^2) \; = \int\limits_{\mu_0^2}^{\mu_F^2} \frac{{\rm d}\mu^2}{\mu^2} \, \frac{1}{x_{\mathbb{P}}} \left[R_g \frac{\alpha_S(\mu^2)}{\mu} x_{\mathbb{P}} g(x_{\mathbb{P}}, \mu^2) \right]^2 \; a^{\mathbb{P}}(z, \mu_F^2; \mu^2). \tag{3}$$

The term $f_{\mathbb{P}}(x_{\mathbb{P}}; \mu^2) \equiv [\ldots]^2/x_{\mathbb{P}}$ plays the role of the Pomeron flux in (2). R_g is the skewed factor which accounts for the fact that in the lower parts of Fig. 1(a,b) we deal not with the diagonal but with the skewed (or generalised) parton distributions. For low values of $x_{\mathbb{P}} \ll 1$ this skewed factor is given by the Shuvaev prescription [9]. The notation $a^{\mathbb{P}}(z, \mu_F^2; \mu^2)$ for the Pomeron parton densities means that they are DGLAP-evolved from an initial scale μ^2 up to the factorisation scale μ_F^2.

At first sight, the integral (3) appears to be concentrated in the infrared region of low μ. However, for DDIS we consider very small $x_{\mathbb{P}}$ values. In this domain, the gluon distribution of the proton has a large anomalous dimension. Asymptotically, as $x_{\mathbb{P}} \to 0$, BFKL predicts $x_{\mathbb{P}} g(x_{\mathbb{P}}, \mu^2) \sim (\mu^2)^{0.5}$ for fixed α_S [10]. In this case the integral (3) takes the logarithmic form, and we cannot neglect the contribution from large scales μ.

By differentiating (3) with respect to $\ln(\mu_F^2)$, the evolution equation now reads [7]

$$\frac{\partial a^{\rm D}(x_{\mathbb{P}}, z, \mu^2)}{\partial \ln \mu^2} \; = \sum_{a'=q,g} P_{a,a'} \otimes a'^{\rm D} \; + \sum_{\mathbb{P}=G,S,GS} P_{a,\mathbb{P}}(z) f_{\mathbb{P}}(x_{\mathbb{P}}; \mu^2)\,. \tag{4}$$

Here the first term, involving the usual parton-to-parton splitting functions, $P_{a,a'}$, arises from DGLAP evolution in the upper parts of Fig. 1(a,c). The second (inhomogeneous) term, involving the Pomeron-to-parton splitting functions, $P_{a,\mathbb{P}}(z) \equiv a^{\mathbb{P}}(z, \mu^2; \mu^2)$, arises from the transition from the two t-channel partons (that is, the Pomeron) to a single t-channel parton in Fig. 1(a). The notation $\mathbb{P} = G, S, GS$ denotes whether the uppermost two t-channel partons are gluons ($\mathbb{P} = G$) or sea-quarks ($\mathbb{P} = S$)[1] , while the interference term is denoted by $\mathbb{P} = GS$. The LO Pomeron-to-parton splitting functions, $P_{a,\mathbb{P}}$, were calculated in [7], where the perturbative Pomeron flux factors, $f_{\mathbb{P}}(x_{\mathbb{P}}; \mu^2)$, are also given.

Simultaneously, we need to add in (1) the direct Pomeron–photon interaction described by the coefficient function $C_{2,\mathbb{P}} = C_{T,\mathbb{P}} + C_{L,\mathbb{P}}$ corresponding to the hard subprocess shown in

[1]Note that besides the two-gluon exchange we also account for the $q\bar{q}$ t-channel colour-singlet state, since even at very small $x_{\mathbb{P}}$ the sea-quark contribution does not die out. Moreover, the skewed factor R_q for the $q\bar{q}$ t-channel state is more than three times larger than that for the gluons [9].

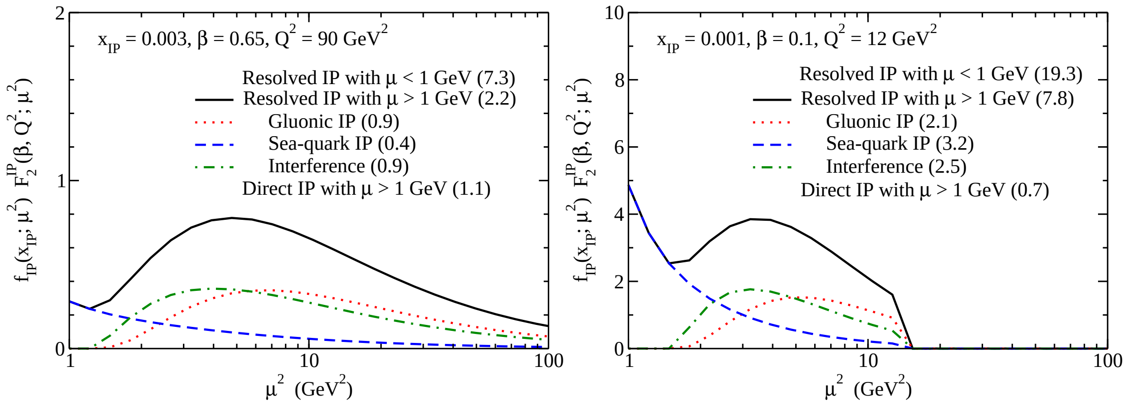

Fig. 2: The breakdown of the resolved Pomeron contribution for $\mu > 1$ GeV to the total $F_2^{\mathrm{D}(3)}$ as a function of μ^2 for two representative values of $(x_{\mathbb{P}}, \beta, Q^2)$. The integral over $\ln(\mu^2)$ is shown by the numbers in parentheses in the legend. Also shown are the integrated contributions from the the resolved Pomeron with $\mu < 1$ GeV and the direct Pomeron with $\mu > 1$ GeV. The secondary Reggeon contributions are negligible for the values of $x_{\mathbb{P}}$ chosen here.

Fig. 1(b):

$$F_2^{\mathrm{D}(3)} \;=\; \sum_{a=q,g} C_{2,a} \otimes a^{\mathrm{D}} \;+\; \sum_{\mathbb{P}=G,S,GS} C_{2,\mathbb{P}}. \tag{5}$$

For the LO $C_{T,\mathbb{P}}$ the light-quark contributions in the limit $\mu^2 \ll Q^2$ are subtracted since they are already included in the first term of (5) via the inhomogeneous evolution of the DPDFs. This subtraction defines a choice of factorisation scheme. There is no such subtraction for the LO $C_{L,\mathbb{P}}$, which are purely higher-twist, or for the heavy quark contributions since we work in the fixed flavour-number scheme (FFNS).

Thus, we see from (4) and (5) that the diffractive structure function is analogous to the photon structure function, where there are both resolved and direct components and where the photon PDFs also satisfy an inhomogeneous evolution equation.

As usual the evolution starts from some not-too-small scale μ_0 and all the contributions, both perturbative and non-perturbative, coming from $\mu \leq \mu_0$ are parameterised in terms of the Regge factorisation as some input which should be fitted to the data. Note that inclusion of the inhomogeneous term in (4) and the direct Pomeron coefficient function in (5) does not add any new free parameters to the description of the DPDFs. The LO Pomeron-to-parton splitting functions are known [7] and at next-to-leading order (NLO) they can be calculated unambiguously. The numerical results [8] presented below were obtained by fitting the H1 LRG data [11] with $M_X > 2$ GeV and $Q^2 \geq 8.5$ GeV^2, adding statistical and systematic experimental errors in quadrature, and taking the input distributions at $\mu_0^2 = 2$ GeV^2, which gave a $\chi^2/\mathrm{d.o.f.}$ of 0.84.

From Fig. 2 we see that integrating the perturbative contributions to $F_2^{\mathrm{D}(3)}$ starting from $\mu = 1$ GeV we collect up to a third of the whole diffractive structure function $F_2^{\mathrm{D}(3)}$. Of course, the numerous corrections (higher order α_S corrections, power corrections, etc.) are not negligible at such low scales as $\mu \sim 1$ GeV. Nevertheless, this fact indicates that an important part of the diffractive parton densities comes from the relatively small size components of the 'Pomeron'.

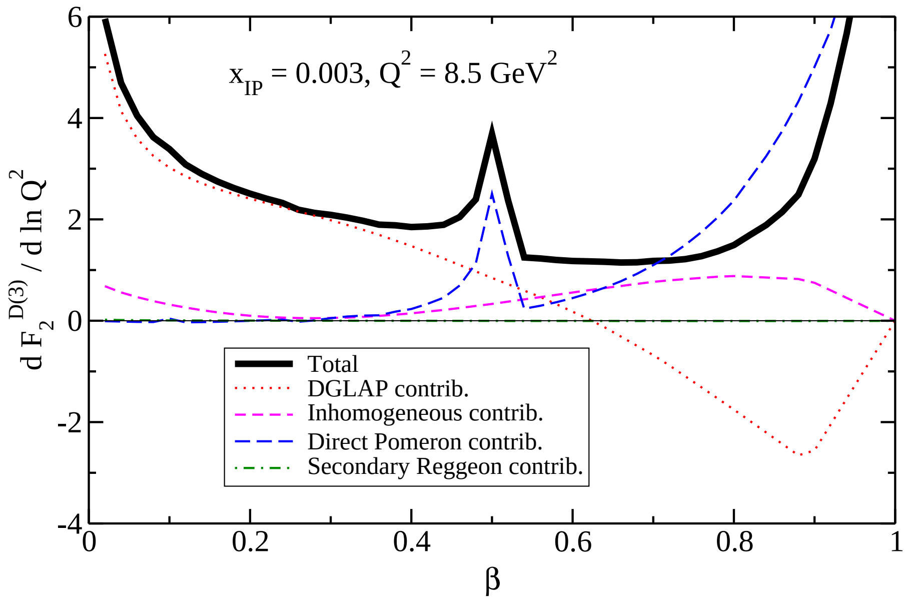

Fig. 3: The breakdown of the contributions to the slope $\partial F_2^{\mathrm{D}(3)}/\partial\ln Q^2$ as a function of β. Contributions apart from the usual DGLAP terms start becoming important for $\beta \gtrsim 0.4$. The peak at $\beta = Q^2/(Q^2+4m_c^2) \simeq 0.51$ is due to the threshold for charm production from the direct Pomeron–photon interaction.

That is why fitting the DDIS data in terms of the Regge factorisation one needs the effective Pomeron intercept $\alpha_{\mathbb{P}}(0) \simeq 1.12$ larger than that ($\alpha_{\mathbb{P}}(0) \simeq 1.08$) measured in the purely soft hadron–hadron scattering.

In Fig. 3 we show the breakdown of the different contributions to the slope $\partial F_2^{\mathrm{D}(3)}/\partial\ln Q^2$. As it is seen, the inhomogeneous term in the evolution equation is fairly large and starts becoming important for $\beta \gtrsim 0.4$. Note that the direct Pomeron contribution at large β, mostly the twist-four $F_L^{\mathrm{D}(3)}$, only gives an important contribution to the total $F_2^{\mathrm{D}(3)}$ for $\beta \gtrsim 0.9$, but the contribution to the Q^2 slope starts becoming important at moderate β. Therefore, it is not possible to avoid the presence of this contribution by simply excluding data points with $\beta > 0.8$ from the fit, as is done in the H1 2006 analysis [11]. In the analysis of inclusive DDIS data, the diffractive gluon density is mainly determined by the derivative $\partial F_2^{\mathrm{D}(3)}/\partial\ln Q^2$. The presence of these additional positive contributions to the Q^2 slope at large β apart from the usual DGLAP contribution means that a smaller gluon density is required for $z \gtrsim 0.4$ compared to the H1 2006 Fit A performed in the usual Regge factorisation framework [11]; see Fig. 4.

In our analysis we account for the twist-four $F_L^{\mathrm{D}(3)}$ contribution coming from the direct Pomeron–photon fusion, that is, from the last term in (5) corresponding to Fig. 1(b). This contribution, which goes to a constant value as $\beta \to 1$, was calculated in [2, 3] and turns out to be numerically appreciable. For the coupling to massless quarks, this contribution to (5) takes the form [7]

$$C_{L,\mathbb{P}} = \int_{\mu_0^2}^{\frac{Q^2}{4\beta}} \mathrm{d}\mu^2\, \frac{1/Q^2}{\sqrt{1-4\beta\mu^2/Q^2}}\, f_{\mathbb{P}}(x_{\mathbb{P}};\mu^2)\, F_L^{\mathbb{P}}(\beta). \tag{6}$$

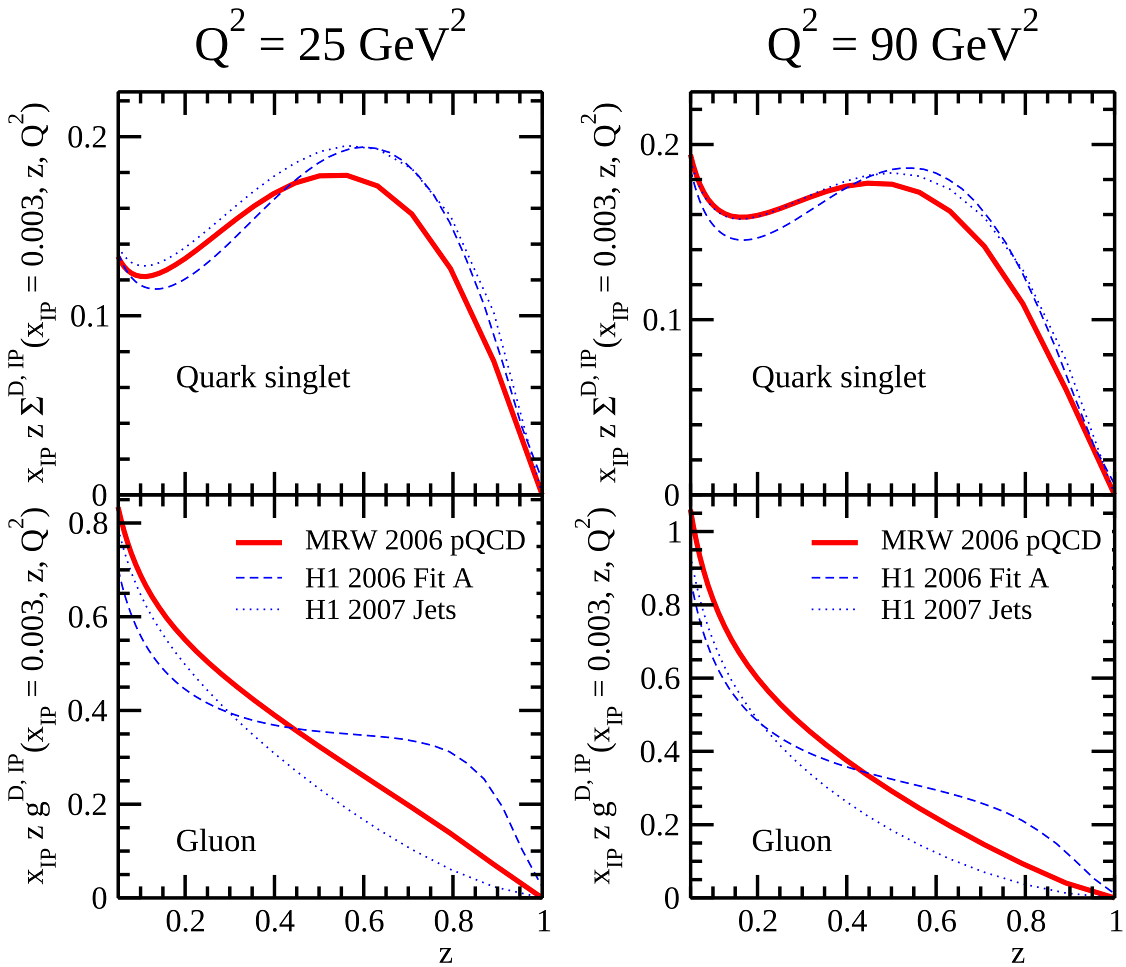

Fig. 4: The MRW 2006 pQCD DPDFs compared to those from the H1 2006 Fit A and the H1 2007 Jets fit. All the DPDFs shown here are normalised to $M_Y < 1.6$ GeV by multiplying by a factor 1.23 relative to $M_Y = m_p$, where M_Y is the mass of the proton dissociative system.

Differentiating with respect to $\ln(Q^2)$ gives a positive term from the integrand evaluated at $\mu^2 = Q^2/(4\beta)$, analogous to the inhomogeneous term in the evolution equation, in addition to the negative term proportional to $-1/Q^2$ obtained from taking the derivative of the integrand.

A recent analysis [12] includes the twist-four $F_L^{\rm D(3)}$ contribution from all k_t in addition to the usual Regge factorisation formulae of (1) and (2). It is found that the twist-four $F_L^{\rm D(3)}$ term gives a negative contribution to the Q^2 slope, such that a larger gluon distribution at high z is needed than in the usual Regge factorisation approach. However, the contribution from $\mu^2 = k_t^2/(1-\beta) < \mu_0^2$ is already included in the input Pomeron PDFs taken at a scale $\mu_0^2 = 1.5$ GeV^2, so some double-counting is involved. Moreover, the DGLAP evolution is not accounted for in the dipole cross section, and the inhomogeneous term in the evolution equation of the DPDFs is neglected. The future measurement of $F_L^{\rm D(3)}$ [13] will provide an important check of

the calculations of the twist-four component.

The smaller diffractive gluon density at high z found in our analysis, compared to the H1 2006 Fit A, is preferred by the data on inclusive diffractive dijet production, $\gamma^* p \to$ dijet $X' + p$, where at LO the dijet system originates from the outgoing $q\bar{q}$ pair in Fig. 1(a,c) and the rest of the hadronic system X' originates from the other outgoing partons in Fig. 1(a,c). However, the data still tend to be slightly overestimated [14]. This can also be seen in Fig. 4 where the 'MRW 2006' gluon at high z is smaller than the 'H1 2006 Fit A' gluon, but larger than the 'H1 2007 Jets' gluon obtained from a combined fit, within the Regge factorisation framework, to inclusive DDIS and inclusive diffractive dijet data [15]. The χ^2 for the 190 inclusive DDIS points increases from 158 (H1 2006 Fit A) to 169 (H1 2007 Jets fit) on inclusion of the dijet data. Therefore, the gluon density determined indirectly from the inclusive DDIS data, under the assumption of pure DGLAP evolution, is different from the gluon density preferred by the dijet data. The apparent tension between the inclusive DDIS and dijet data in the Regge factorisation approach is partly alleviated by the inclusion of the perturbative Pomeron terms.

Note that in the 'MRW 2006' analysis [8] the parton distributions of the proton, representing the lower parton ladders in Fig. 1(a,b), were taken from a NLO global fit. However, since the Pomeron-to-parton splitting is only calculated at LO, it may be more appropriate to take the inclusive parton distributions from a LO fit, where the gluon density is much larger at small x and does not take a valence-like form at low scales. In this case, the inhomogeneous term in the evolution equation, and the other direct Pomeron contributions, would be enhanced, leading to an even smaller diffractive gluon density at high z. Note that even if LO DGLAP evolution is used in the lower parton ladders of Fig. 1(a,b), NLO DGLAP evolution may still be used for the evolution of the Pomeron PDFs in the upper parts of Fig. 1(a,c).

A more direct way to observe the perturbative Pomeron contribution to DDIS is to study the transverse momenta of secondaries in the 'Pomeron fragmentation' region, at the edge of the LRG, as in Ref. [16]. In contrast with the Regge factorisation of Fig. 1(c), where the transverse momentum distribution of the partons inside a Reggeon is assumed to have an exponential form with a rather low mean value of intrinsic k_t, in the perturbative case of Fig. 1(a) the k_t-distribution of the lowest jet in Fig.1(a) obeys a power law, given by the integrand of (3). Therefore we expect a larger k_t of the secondaries with the long power-like tail.

So far, measurements of inclusive diffractive dijets at HERA have primarily been made in the kinematic region of small $\beta \equiv x_B/x_{\mathbb{P}}$ where the resolved Pomeron mechanism of Fig. 1(a,c) gives the dominant contribution, and the contribution from exclusive diffractive dijets, Fig. 1(b), is negligible. However, a first measurement has been made of dijets in DDIS with a cut on $\beta > 0.45$ in order to enhance the contribution from exclusive diffractive dijets, $\gamma^* p \to$ dijet $+\, p$ [17]. Within the HERA kinematic domain the sea-quark component of the Pomeron is quite important (see Fig. 2). This statement can be checked by observing the diffractive high-E_T dijet distributions in the exclusive $\gamma^{(*)} p \to$ dijet $+\, p$ process. At LO we expect the ratio of cross sections for high-E_T dijets produced via the two-gluon (gg) and $q\bar{q}$ t-channel exchange to be [7]

$$\frac{\left.(\mathrm{d}\sigma_T^{\gamma^* p}/\mathrm{d}E_T)\right|_{\mathbb{P}=G}}{\left.(\mathrm{d}\sigma_T^{\gamma^* p}/\mathrm{d}E_T)\right|_{\mathbb{P}=S}} = \frac{81}{4}\left[\frac{\alpha(1-\alpha)Q^2}{E_T^2+\alpha(1-\alpha)Q^2}\,\frac{R_g\, x_{\mathbb{P}} g(x_{\mathbb{P}},\mu^2)}{R_q\, x_{\mathbb{P}} S(x_{\mathbb{P}},\mu^2)}\right]^2, \tag{7}$$

where α is the photon's light-cone momentum fraction carried by the high-E_T jet, and the scale $\mu^2 = \alpha(1-\alpha)Q^2 + E_T^2$. Therefore, measurements of exclusive diffractive dijets at large E_T in photoproduction would probe the presence of the $q\bar{q}$ t-channel exchange. However, it remains to be seen whether a measurement of exclusive diffractive dijets is feasible without imposing a cut on β.

In summary, we have shown how to obtain *universal* diffractive parton densities $a^{\rm D}$ which can be used in the description of different diffractive processes. We emphasise that the perturbative QCD contribution originated by the inhomogeneous (last) term in (4) is not small and, starting from a relatively low scale $\mu_0 = 1\,$GeV, may generate up to a third of the diffractive parton densities. Simultaneously, an account of this inhomogeneous term, and the twist-four $F_L^{\rm D(3)}$ contribution, leads to a lower diffractive gluon density at high z in comparison to that obtained using the Regge factorisation hypothesis. The presence of the perturbative (large scale) contributions reveals itself in larger transverse momenta (with a power-like tail) of the secondaries observed in the 'Pomeron fragmentation' region. The colour singlet sea-quark pair exchange is an important component of the perturbative Pomeron, which plays the dominant role in exclusive diffractive dijet production with $E_T^2 \gg Q^2$. Parameterisations for the DPDFs and the diffractive structure functions are made publically available.[2]

Acknowledgments

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